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Modelling Golden Gate Bridge

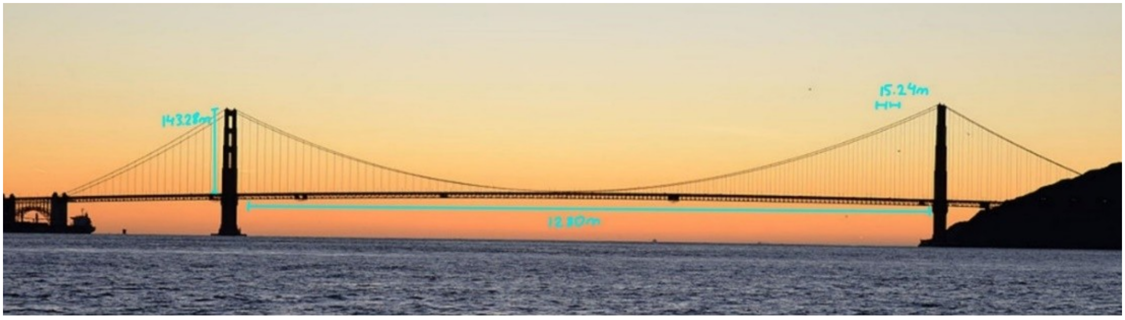

Figure 1: Golden Gate Bridge panorama taken by D Ramey Logan (2015)

Introduction

The Golden Gate Bridge is a significant American landmark connecting California to the rest of USA. It is famous for its bright red paint coating which must be maintained to uphold the iconic appearance (Britannica, 2022). The aim of this investigation was to find the total surface area of vertical wires suspending the bridge from the parabolic arc (excludes wire outside of the arc) in order to predict the surface area of red paint needed to repaint the wires for the bridge. The following is a table of the relevant dimensions for the investigation. The reference image used is in Figure 1

Table 1: Variables for Golden Gate Bridge provided by Golden Gate Bridge Highway & Transportation District (2019)

Dimension/variable	Actual Value (m)
Length (l)	1280
Height ($h¿$	143.3
Distance between wires	15
Diameter of wires $(2r)$	1
Surface area of wire (SA)	?

Plotting Points

The image in figure 1 was imported into Desmos. The two towers were aligned to $l=0$ and $h=1280$ and dragged to $y=143.3$. This was done to match the dimensions of the bridge to a 1:1 scale where 1m unit on the graph is equal to 1 metre for the actual bridge. The transparency was reduced for the points to have higher visibility. The domain and range for the plotted points was restricted to the dimensions of the bridge in Table 1 for there to not be an excess of points that do not represent the bridge.

$D:\{0 \leq l \leq 1280\} R:\{0 \leq h \leq 143.3\}$

Points plotted and dragged at metric 100m intervals (except for final point) along the translucent image of the bridge. The dragging of these points was done manually and based on visual judgement, thus decreasing the degree of precision for the h values. The resolution of the reference image in Figure 1 was also a limitation for the degree of precision because the points were placed in the centre of the pixels on the arc. This may not have been representative of the bridge because the wires were attached to the bottom of the wire for the arc. Additionally, the photograph may be subject to distortion which would limit the accuracy of the model to how discrepant it is from the potentially distorted reference image, and not the actual bridge. This did not affect the l values because these intervals were standardised and even. The 100m interval was chosen because it shows the change in the height as the length increases by 100m at a time. The following table of co-ordinates was constructed.

Table 2: co-ordinates of points table

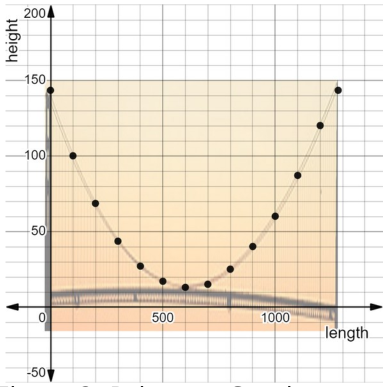

Figure 2: Points on Graph

x_1 / l	y_1 / h
0	143.3
100	100
200	68.5
300	43.5
400	27
500	17
600	13
700	15
800	25
900	40
1000	60
1100	87
1200	120
1280	143.3

Exponential Model

Because the bridge in Figure 1 resembled two exponential graphs because of the flattening in the middle of the arc, the points in table 2 from where $x_1 = 0$ to $x_1 = 600$ were used to model the arc until the turning point on an Excel graph. The second half used the points $x_1 = 600$ and above. The combined graph produced in Excel is in Appendix 1.
From the excel exponential trend line in Figure 3, the equations for the model were:

$$h \approx 148.83\, e^{-0.004 l} \{0 \le l \le 600\}, R^2 = 0.99668 \quad (1)$$
$$h \approx 1.2352\, e^{0.0038 l} \{600 \le l \le 1280\}, R^2 = 0.9849 \quad (2)$$

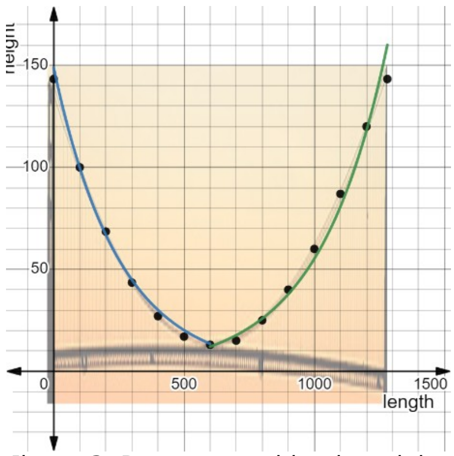

Figure 3: Desmos combined model of exponential Golden Gate Bridge

Although the line fitted the points well up until $l \le 600$, above that, there were large residuals for the second half of the model with the largest one being 16.71m from its point. These large residuals would greatly increase the error when calculating the vertical lengths for the wires because the arc would be distorted and nonrepresentative of the bridge arc. Because the length of the wire is directly proportional to the surface area of it, this error would be magnified and increase uncertainty in the final total surface area. Another weakness of this model is the visual unfitness against the reference image. The two exponential function's intersections created a pointed tip at $l = 600$ which did not visually fit the reference image's curved shape. Additionally, the two exponential graphs do not intersect and thus the model is incomplete and is not representative of the real-life bridge. Because of these flaws, the exponential method was eliminated for modelling the Golden Gate Bridge.

Quadratic Model using Matrices

Therefore, the next approach was modelling a quadratic function because the bridge had 1 turning point in the centre of the bridge, appears symmetrical, and resembled a concave up quadratic.

The matrices method was used in place of the quadratic regression model to find the line equation. This was done by selecting 3 points and using a 3x3 matrix. The 3 points chosen were:

$(l_1, h_1) = (0, 143.3)(l_2, h_2) = (600, 13)(l_3, h_3) = (1280, 143.3)$

These were chosen because the key components of a quadratic are the maximum and minimum points, axis intercepts, and turning point – all of which are covered in these 3 points.

$$\begin{bmatrix} l_1^2 & l_1 & 1 \\ l_2^2 & l_2 & 1 \\ l_3^2 & l_3 & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} l_1^2 & l_1 & 1 \\ l_2^2 & l_2 & 1 \\ l_3^2 & l_3 & 1 \end{bmatrix} \times \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} l_1^2 & l_1 & 1 \\ l_2^2 & l_2 & 1 \\ l_3^2 & l_3 & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} h_1 \\ h_2 \\ h_3 \end{bmatrix}$$

$$\begin{bmatrix} 0 & 0 & 1 \\ 360{,}000 & 600 & 1 \\ 1{,}638{,}400 & 1280 & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} 0 & 0 & 1 \\ 360{,}000 & 600 & 1 \\ 1{,}638{,}400 & 1280 & 1 \end{bmatrix} \times \begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0 & 0 & 1 \\ 360{,}000 & 600 & 1 \\ 1{,}638{,}400 & 1280 & 1 \end{bmatrix}^{-1} \times \begin{bmatrix} 143.3 \\ 13 \\ 143.3 \end{bmatrix}$$

$$\begin{bmatrix} a \\ b \\ c \end{bmatrix} = \begin{bmatrix} 0.000319 \\ -0.408784 \\ 143.3 \end{bmatrix}$$

From this matrix, the following quadratic equation was constructed.

$$h = 0.000319\, l^2 - 0.408784\, l + 143.3 \{0 \le l \le 1280\} h \approx 0.00319\, l^2 - 0.409\, l + 143 \{0 \le l \le 1280\}$$

(3)

Because 3 points were insufficient for calculating an R^2 value, the co-efficient determination was calculated by producing a graph using the equation above and comparing the R^2 to the points in Table 2 in Desmos. The following coefficient of determination and graph (figure 3) was produced.

$R^2 = 0.9879$

Although this method created a model which predicted and fit the shape of the actual bridge to 98.79%, the accuracy to the actual bridge can be increased by also producing a quadratic but using more points. This will increase the accuracy because by increasing the number of data points, the sample size is increased which thus decreases the margin of error for the data (Bartlett, J. et al., 2001). By doing so, this increases the accuracy to the actual bridge.
Therefore, quadratic regression was used with all 14 data points in Table 2 to produce the final model.

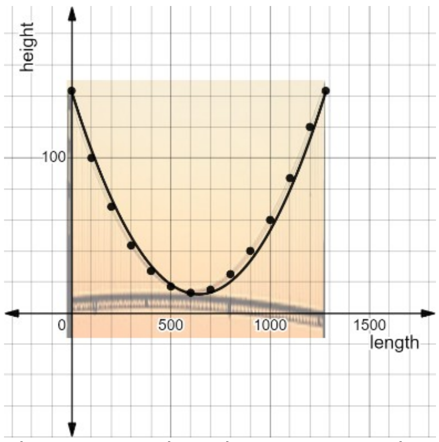

Figure 4: Produced Desmos graph for matrix quadratic

Quadratic Model using Quadratic Regression

The points in Table 2 were used to produce a quadratic function using the equation below.

$$y_1 \;\; a x_1^2 + b x_1 + c \{0 \le x_1 \le 1280\}$$

From this, Desmos produced the following values in Table 3. From these values, the quadratic equation was be constructed, and the model in Figure 5 was graphed. The a, b, c values were rounded to 3 significant figures to keep consistent with the previous model's degree of precision.

(4)

$$h = 0.000318314\, l^2 - 0.398304\, l + 137.76 \left(0 \le l \le 1280\right)$$
$$h \approx 0.000318\, l^2 - 0.398\, l + 137 \left(0 \le l \le 1280\right)$$

Figure 5 (right): quadratic model produced

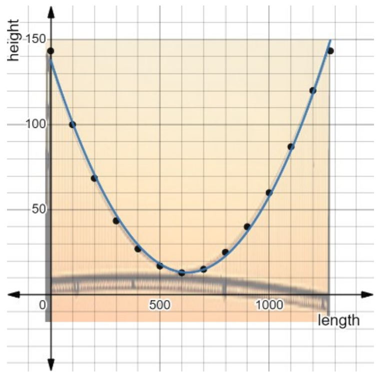

Table 3 (below): produced values from Desmos for quadratic regression

Variable	Produced value
R^2	0.9958
a	0.000318314
b	-0.398304
c	137.76

The coefficient of determination was kept as the actual value from Desmos and not rounded to 3 significant figures because the other R^2 values would not have a significant difference at 3 significant figures. Therefore, by using the exact value from Desmos for the coefficients of determination, the two quadratic models were distinct and could be compared against each other. The co-efficient of determination for the exponential model was not considered because this model was eliminated earlier in the investigation.

Table 4: coefficients of determination for quadratic models.

Model	Coefficient of Determination
Quadratic using matrices	0.9879
Quadratic using quadratic regression	0.9958

Although the coefficient of determination was near 1, which indicates high precision and low random error, the quadratic regression model was slightly offset from the image on both sides of the turning point. However, this slight discrepancy from the actual model can be dismissed because of the high coefficient of determination value which indicates that the residuals are insignificant, and that the variation of the height can explain the variation of the length to 99.58%. Because this value is greater than that of the previous quadratic model and more data points were used, this quadratic regression model was concluded to be the best representation for the arc of Golden Gate Bridge. Therefore, this function was used for finding the surface area for the 85 wires.

Surface Area of Wires

The lengths of the wires were found by finding the h values for every 15m interval in the l because this was the given distance between the wires. This was a sample calculation for the length of the first wire at $l = 15$ on the bridge. \The length was substituted into the function equation for the quadratic model using quadratic regression (Equation 4). The unrounded value for h was carried through to the following step to reduce error.

$$h = 0.000318\, l^2 - 0.398\, l + 137 \text{¿} 0.000318(15)^2 - 0.398(15) + 137 \approx 131.857\, m$$

This substitution was repeated by computer on Excel. Afterwards, the sum of the lengths of the wires was calculated using the $\sum$ ¿ excel function. Again, the

unrounded value was used for the following step, which finds the total surface area, to reduce error. This function produced the following value:

$$\sum h \approx 4796.5398\, m$$

The sum of these lengths was the total wire length for the bridge arc. From this, the total surface area was found using surface area of a cylinder. The wires were assumed to be at $15 \leq l \leq 1,275$ because at $l = 0$ was one of the towers and there can only be 85 wires therefore the final wire will be at $l = 1275$ instead of 1280.

$$\sum SA = 2\pi r\left(\sum h\right) + 2\pi r^2 ¿ 2\pi\left(\frac{1}{2}\right)(4796.5398) + 2\pi\left(\frac{1}{2}\right)^2 \approx 15070.3 \approx 1.51 \times 10^4\, m^2$$

Conclusion

In this investigation, the Golden Gate Bridge was modelled using 3 methods – combined exponential, matrix quadratic, quadratic regression. The most effective was the quadratic regression model because it was a complete model, had the highest co-efficient of determination, visually fit the reference image, and had the basis of 14 data points. Using this model, the total surface area of the wires that need to be repainted on the bridge was determined to be:

$$\sum SA \approx 1.51 \times 10^4\, m^2$$

Bibliography

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Appendix

Appendix 1: Combined exponential model of Golden Gate Bridge by author